Factorization of Lepton Radiation in SIDIS

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Proton Spin Question

- Total spin decomposed as in (Jaffe and Manohar, 1990; Ji, 1997)

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

- Sum of quark spins do not match proton spin

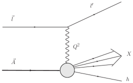
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

- Polarized DIS experiments complementary to RHIC to determine ΔG (currently ≈ 0.4)
- $L_{q/g}$ related to transverse motion $\Rightarrow$ TMDs

JLab, EIC, and SIDIS

- Current Jefferson Lab (JLab) 12 GeV and upcoming Electron Ion Collider (EIC) will perform electron-proton scattering to probe structure of proton
- Structure probed in semi-inclusive deep inelastic scattering (SIDIS) is related to parton distribution functions (PDFs) and transverse momentum dependent PDFs (TMDs)
- Factorization connects measured hadron to PDFs and TMDs
- Kinematics defined in photon-hadron frame under the one-photon approximation

$$\mathrm{d}\sigma \propto \left| \quad \right|^2 \propto L^{\mu\nu}(q,\ell)W_{\mu\nu}(q,p,\ldots)$$

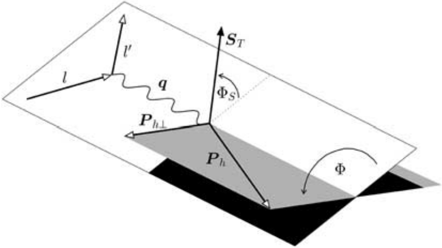


- Collinear factorization separates hadronic portion into two parts: PDFs and fragmentation functions (FFs)

$$E_h E_{\ell'} \frac{\mathrm{d}^3\sigma^{\ell A\to\ell' hX}}{\mathrm{d}^3\vec{P}_h\,\mathrm{d}^3\vec{\ell'}} = \frac{1}{2S}\sum_{i,j}\int \frac{\mathrm{d}x}{x}\frac{\mathrm{d}z}{z^2}\tilde{f}^{i/A}(x)\tilde{D}^{h/j}(z)\hat{\sigma}^{ei\to e'j}(x,z) \qquad (1)$$

Photon-Hadron Frame

- Standard definitions uses Trento Convention (Bacchetta et al., 2004)



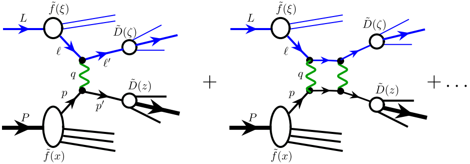
- Radiation of photon from initial state changes the plane modulation
- Unobserved lepton radiation introduces ambiguity, as the radiation changes the unobserved photon's momentum (Liu et al., 2021)
- Unified factorization method to address these issues

New Factorization Approach

- Unified factorization scheme (QED and QCD on equal footing) (Liu et al., 2021)
- Leading collinear radiative corrections resummed into lepton distribution functions and lepton fragmentation functions
- Focus on collinear regime at first

$$E_h E_{L'} \frac{\mathrm{d}^3\sigma^{LA\to L'hX}}{\mathrm{d}^3\vec{P}_h\,\mathrm{d}^3\vec{L'}} = \frac{1}{2S}\sum_{i,j,n,m}\int \frac{\mathrm{d}\zeta}{\zeta^2}\frac{\mathrm{d}\xi}{\xi}\frac{\mathrm{d}z}{z^2}\frac{\mathrm{d}x}{x}\tilde{D}_{m/e}(\zeta)\tilde{f}_{e/i}(\xi) \times \tilde{D}_{h/n}(z)\tilde{f}_{j/h}(x)\hat{H}^{ij\to nm}(\xi,\zeta,x,z) \quad (2)$$

- Note no structure functions in this expression (which is valid beyond the one-photon exchange)
- Can still extract PDFs and TMDs from scattering and can naturally take care of higher order QED contributions



Lepton-Hadron (Lab) Frame

- Since the direction of the virtual photon is unknown, two common kinematic frames chosen
- Defining the external momenta in the lab frame using the rapidity and transverse momenta,

$$L^{\mu} = \left(\sqrt{\frac{S}{2}}, 0^{-}, \vec{0}_{\perp}\right), \quad L'^{\mu} = \left(\frac{L'_T}{\sqrt{2}} e^{y_L}, \frac{L'_T}{\sqrt{2}} e^{-y_L}, \vec{L}'_T\right)$$

$$P^{\mu} = \left(0^{+}, \sqrt{\frac{S}{2}}, \vec{0}_{\perp}\right), \quad P'^{\mu} = \left(\frac{P'_T}{\sqrt{2}} e^{y_P}, \frac{P'_T}{\sqrt{2}} e^{-y_P}, \vec{P}'_T\right)$$

where

$$L'_T = \left|\vec{L}'_T\right|, \quad P'_T = \left|\vec{P}'_T\right|, \quad y_L = \frac{1}{2}\log\left(\frac{L'^{+}}{L'^{-}}\right), \quad y_P = \frac{1}{2}\log\left(\frac{P'^{+}}{P'^{-}}\right)$$

Virtual Photon-Hadron Frame

- Traditionally Breit frame is photon-hadron frame
- Lepton radiation makes frame determination ambiguous
- All historical factorization formula defined in photon hadrone frame
- Introduce **virtual** photon-hadron frame which is determined by a given pair of ξ, ζ under one-photon exchange approximation

$$x_B = \frac{Q^2}{2P\cdot q} \quad \Rightarrow \quad \hat{x}_B = \frac{\hat{Q}^2}{2P\cdot \hat{q}}$$

$$y = \frac{P\cdot q}{P\cdot L} \quad \Rightarrow \quad \hat{y} = \frac{P\cdot \hat{q}}{P\cdot L}$$

$$z_h = \frac{P\cdot P'}{P\cdot q} \quad \Rightarrow \quad \hat{z}_h = \frac{P\cdot P'}{P\cdot \hat{q}}$$

Transformation Between Frames

- Given generic vector in Breit frame

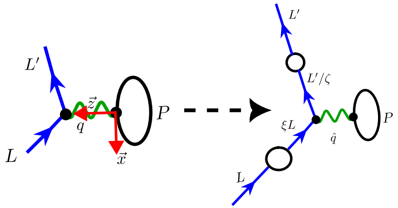
$$x^\mu = \left(x^+, x^-, \vec{x}_\perp = (x^1, x^2)\right)$$

- Lorentz boost (ϕ) to hadron rest frame then rotate ($\theta(\xi,\zeta)$)around $y-$axis ($\perp$ to lepton plane) so γ along $z-$axis

$$\tilde{x}^\sigma = R^\sigma_\nu \Lambda^\nu_\mu x^\mu$$

$$y \Rightarrow \tilde{y} = y$$

$$\begin{pmatrix} x \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{x} \\ \tilde{z} \end{pmatrix} = R(\theta(\xi,\zeta)) \begin{pmatrix} x \\ z \end{pmatrix}$$



Helicity Basis

- Vectors defined similar to (Ji et al., 2006)
- Work with helicity-based hadronic structure functions (similar to lepton case in (Liu et al., 2021))

$$\hat{W}^{\mu\nu} = \frac{1}{2}(\underbrace{\hat{X}^\mu \hat{X}^\nu + \hat{Y}^\mu \hat{Y}^\nu}_{\text{Transverse}})\hat{H}_{TT} + (\underbrace{\hat{T}^\mu \hat{T}^\nu \hat{H}_L + \left(\hat{T}^\mu \hat{X}^\nu + \hat{X}^\mu \hat{T}^\nu\right)\hat{H}_\Delta + \ldots}_{\text{other, } \mathcal{O}\left(\alpha_S^2\right)})$$

$$\hat{Z}^\mu = -\frac{\hat{q}^\mu}{\hat{Q}}$$

$$\hat{T}^\mu = \left(\frac{1}{\hat{Q}}\right)\left(\hat{q}^\mu + 2\hat{x}_B P^\mu\right)$$

$$\hat{X}^\mu = \left(\frac{1}{\hat{\vec{q}}_T}\right)\left(\frac{(P')^\mu}{\hat{z}_h} - \hat{q}^\mu - \left(1 + \frac{\hat{\vec{q}}_T^2}{\hat{Q}^2}\right)\hat{x}_B P^\mu\right)$$

$$\hat{Y}^\mu = \epsilon^{\mu\nu\alpha\beta}\hat{Z}_\nu \hat{T}_\alpha \hat{X}_\beta$$

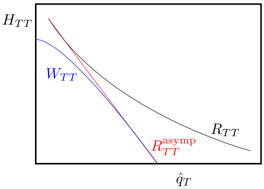
Structure Functions

- For the fixed order calculation, where $W_{\mu\nu}$ projects the hadron to the partonic states,

$$H_{TT} = \frac{1}{2}(X^\mu X^\nu + Y^\mu Y^\nu)W_{\mu\nu}$$

- where the real and virtual parts as power series of α_S form $W_{\mu\nu} = R_{\mu\nu} + V_{\mu\nu}$
- These also depend on $\vec{\hat{q}}_T$:
  - When $\vec{\hat{q}}_T \sim \hat{Q}$, fixed order is under control (no large logarithms)
  - when $\vec{\hat{q}}_T >> \hat{Q}$ or $\vec{\hat{q}}_T << \hat{Q}$, large logarithm divergences (focus on latter)

$$H_{TT}(\vec{\hat{q}}_T) = W_{TT}(\text{small } \vec{\hat{q}}_T) + Y_{TT}(\text{large } \vec{\hat{q}}_T)$$
$$Y_{TT} = R_{TT} - R_{TT}^{\text{asymp}}$$



CSS-like Formalism

- Better to work in Fourier Transform of $\vec{q}_T$ to $\vec{b}_T$ space (as in the CSS formalism (Collins et al., 1985))

$$\tilde{W}_{TT}(\vec{b}_T, Q) = \int \mathrm{d}^2\vec{q}_T e^{i\vec{q}_T\cdot\vec{b}_T} W_{TT}(\vec{q}_T, Q) = e^{-S}[C_f \otimes f] \otimes [C_D \otimes D]$$

- Expand resummed expression for $\tilde{W}_{TT}(\vec{b}_T)$ up $\mathcal{O}(\alpha_S)$

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

where

$$S = 1 - \frac{\alpha}{\pi}\left[\frac{1}{2}A^{(1)} \ln^2 \frac{\nu_Q^2}{\mu_b^2} + B^{(1)} \ln \frac{\nu_Q^2}{\mu_b^2}\right]$$

- Important for defining hadronic structure function, which are related to PDFs

NLO Perturbative Coefficients

$$\tilde{W}_{TT} = \frac{4e_Q^2\alpha_S}{3(2\pi)}\left[\left(\left(\frac{2\lambda^2-2\lambda+1}{\lambda}\right)\frac{\delta(1-\eta)}{(1-\lambda)_+}+\left(\frac{2\eta^2-2\eta+1}{\eta}\right)\frac{\delta(1-\lambda)}{(1-\eta)_+}\right)\right.$$
$$+\left(\left(\frac{\lambda^2+1}{\lambda}\right)\frac{\delta(1-\eta)}{(1-\lambda)_+}+\left(\frac{\eta^2+1}{\eta}\right)\frac{\delta(1-\lambda)}{(1-\eta)_+}\right)\left[-\frac{1}{\epsilon}-\ln\frac{\mu_{\bar{M}S}^2}{\mu_b^2}\right]$$
$$\left.+2\delta(1-\lambda)\delta(1-\eta)\left(-\frac{2}{\epsilon}+\frac{1}{2}\left(-\ln^2\frac{\mu_b^2}{\nu_Q^2}-3\ln\frac{\mu_b^2}{\nu_Q^2}-3\ln\frac{\mu_{\bar{M}S}^2}{\mu_b^2}-4\right)\right)\right] \tag{3}$$

$$A^{(1)} = 1 \tag{4}$$

$$B^{(1)} = -\frac{3}{2} \tag{5}$$

$$C_f^{(1)}(\lambda) = \frac{1}{2\lambda}(1-2\lambda)-\frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+\ln\frac{\mu_{\bar{M}S}}{\mu_b}-\delta(1-\lambda) \tag{6}$$

$$C_D^{(1)}(\eta) = \frac{1}{2\eta}(1-2\eta)-\frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+\ln\frac{\mu_{\bar{M}S}}{\mu_b}-\delta(1-\eta) \tag{7}$$

Conclusions

- Applied joint factorization scheme for SIDIS process
- Derived general Lorentz transformation between virtual photon-hadron frame and lepton-hadron lab frame (or Lab-Breit frame)
- Derived perturbative coefficients in the CSS formalism in the virtual photon-hadron frame
- Next Steps:
  - Prepare code to make predictions for unpolarized and polarized PDFs from SIDIS in CSS approach
  - Expand to study the TMD approach
  - Use known PDFs to test sensitivities of ΔG on q and G at JLab 12 and EIC

Acknowledgements

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